HW #5

1. Consider a 2-sphere as described by the coordinates (r, ϕ) discussed in class.

a) Show that the circumference C is given by $C = 2\pi R \sin(r/R)$.

b) Show that the area of a circle of radius r is given by $A = 2\pi R^2[1 - \cos(r/R)]$.

c) Show that these expressions take their usual values for a flat surface when $R \to \infty$.

2. Consider a 2-dimensional surface of a cylinder of radius R. For coordinates, choose conventional cylindrical coordinates with the z axis running along the axis of the cylinder. The coordinates for a point on the surface will thus be (z, ϕ).

a) Determine the elements of the metric tensor.

b) Use the geodesic equation to determine the parametric equation of a geodesic for this surface.

c) What type of curve is the geodesic in part b)? Discuss the geometry of this surface. Is it the same as that of a flat 2-dimensional space?

(3.) Consider an orthogonal coordinate system such that $g_{ab} = 0$ for $a \neq b$.

a) Show that for a 2-dimensional surface the element of area is given by $dA = \sqrt{g_{11}g_{22}}dx^1dx^2$.

b) Show that the volume element in a 3-dimensional space is given by $dV = \sqrt{g_{11}g_{22}g_{33}}dx^1dx^2dx^3$.

c) Show that your results reduce to the familiar expressions for dA in 2-dimensional polar coordinates and dV in 3-dimensional spherical coordinates.