QUANTUM MECHANICS A (PHY–5645)

HOMEWORK 7

(October 18, 2016)

Due on Tuesday, October 25, 2016

PROBLEM 19

We have solved in class the problem of a particle of mass m confined to a one-dimensional *symmetric* infinite square well of size L. Now consider the same problem but for the *asymmetric* case in which the infinite square well potentials is instead defined as:

$$V(x) = \begin{cases} 0 & \text{if } 0 < x < L, \\ \infty & \text{if } x \leq 0 \text{ or } x \geq L. \end{cases}$$

(a) Obtain all the eigenvalues and all the eigenvectors of the Hamiltonian for the above potential. Are there any differences between the energies obtained in this case relative to the one we solved in class?

(b) Obtain the expectation (or average) values of $\langle \hat{X} \rangle$ and $\langle \hat{P} \rangle$ as well as their corresponding uncertainties ΔX and ΔP assuming that the particle is in the ground state. What value do you obtain for the product $\Delta X \cdot \Delta P$?

(c) Repeat part (b) but now assuming that the particle occupies the first excited state.

PROBLEM 20

The attractive one-dimensional "Delta function potential" is defined as

$$V(x) = -\alpha\delta(x),$$

where $\alpha > 0$ and $\delta(x)$ is the the Dirac delta function.

(a) Find the unique bound state energy and corresponding normalized wave function of the Delta function potential. Is a minimum value of α required in order for the bound state to exist? Note that although the wave function is continuous at $x = 0$, its derivative is not. Make a simple sketch of the wave function.

(b) Obtain the normalized wave function in the *momentum basis.*

(c) Determine the product of the uncertainties $\Delta X \cdot \Delta P$.

PROBLEM 21

Consider a particle of mass m moving in a one-dimensional attractive square-well potential of strength $V_0 > 0$ and size L. That is,

$$V(x) = \begin{cases} -V_0 & \text{if } |x| < L/2, \\ 0 & \text{if } |x| \geq L/2. \end{cases}$$

Given that the potential is invariant under a parity transformation, the eigenstates of the Hamiltonian may be classified according to their parity. That is, even under parity $[\varphi(-x)=\varphi(x)]$ or odd under parity $[\varphi(-x)=-\varphi(x)]$. We have solved in class for the even-parity case; in this problem you will solve the corresponding problem for the odd-parity wave functions.

(a) Show that in the present case, there is a minimum value that the dimensionless quantity $R^2 \equiv mV_0L^2/2\hbar^2$ must attain in order for at least one odd-parity state to be bound.

(b) Obtain the minimum value that the dimensionless quantity R^2 must attain in order for *two* odd-parity states to be bound.

(c) Using the minimum value of R^2 found in part (b), compute the energy and corresponding normalized wave function of the lowest energy odd-parity state.

(d) For the wave function found in part (c), obtain the expectation value of the kinetic energy $\langle T \rangle$, the expectation value of the the potential energy $\langle V \rangle$, and the probability of finding the particle in the classically forbidden region.