QUANTUM MECHANICS B (PHY–5646)

HOMEWORK 22

(April 11, 2017)
Due on Tuesday, April 25, 2017

PROBLEM 64

Consider a particle of mass m and energy $E > 0$ that scatters from a spherically symmetric Yukawa potential of the form

$$V(r) = V_0 \frac{e^{-\mu r}}{r},$$

where $\mu > 0$ defines the range of the potential.

(a) Compute the scattering amplitude $f(\theta)$ using the first Born approximation.

(b) Compute the differential cross section using the first Born approximation; we will show in class that the differential cross section is simply given by:

$$\frac{d\sigma}{d\Omega} = \Big| f(\theta) \Big|^2.$$

(c) By taking suitable limits, compute the *Coulomb cross section* and compare it with the classical limit, which was evaluated for the first time by Rutherford. It turns out that the Coulomb cross section in the Born approximation is exact in both the classical and quantum limits.

PROBLEM 65

Consider a particle of mass m and energy $E > 0$ that scatters from a spherically symmetric attractive square-well potential of strength $V_0 > 0$ and size R. That is,

$$V(r) = \begin{cases} -V_0 & \text{if } 0 < r < R, \\ 0 & \text{if } r > R. \end{cases}$$

(a) Compute the scattering phase shift $\delta_l(E)$ for all values of l. Note that you may leave the answer in terms of spherical Bessel and Neumann functions.

(b) Provide an explicit answer for the s-wave phase shift $\tan \delta_0(E)$ in terms of simple trigonometric functions.

(c) Imagine that the energy E and angular momentum l are such that the classical turning point $r_{\rm cl}$ is outside the range of the potential R. Without doing any calculations, estimate the value of the phase shift $\delta_l(E)$ in the limit of $r_{\rm cl} \gg R$.

PROBLEM 66

The scattering length "a" is obtained from the low-energy behavior of the phase shift. In particular, for the case of $l\!=\!0$ it is defined as follows:

$$-\frac{1}{a} = \lim_{k\to 0} k\cot\delta_0(k) \quad \text{where} \quad k = \sqrt{\frac{2mE}{\hbar^2}}.$$

(a) Compute the s-wave scattering length in units of R (*i.e.*, a/R) for the attractive square-well potential defined in Problem 65.

(b) Make a plot of a/R in terms of the dimensionless quantity ξ defined as

$$\xi \equiv \sqrt{\frac{2mV_0R^2}{\hbar^2}}.$$

In particular, show that the scattering length diverges at $\xi\!=\!\pi/2, 3\pi/2, 5\pi/2, \ldots$

(c) From the perspective of the *bound-state problem*, what is the relevance (if any) of the values $\xi\!=\!\pi/2, 3\pi/2, 5\pi/2, \ldots$