QUANTUM MECHANICS B (PHY–5646)

HOMEWORK 21

(March 31, 2017)
Due on Tuesday, April 11, 2017

PROBLEM 61

A particle of mass m is initially (*i.e.*, at time $t=0$) in the ground state of a one dimensional harmonic oscillator potential of frequency ω. At exactly $t=0$ a perturbation of the form $\hat{H}_1=\lambda\, e^{-t/\tau}\hat{X}^3$ is turned on. Calculate to first order in λ the probability that after a sufficiently long time (i.e., as $t\rightarrow\infty$) the particle would have made a transition to a given excited state of the harmonic oscillator.

PROBLEM 62

A particle of mass m in the infinite past (*i.e.*, at time $t=-\infty$) is occupying the ground state of an infinite potential well whose walls are located at $x=0$ and at $x=a$. At exactly $t=-\infty$ a perturbation of the form $\hat{H}_1=\lambda\, e^{-t^2/\tau^2}\hat{X}$ is turned on. Calculate to first order in λ the probability that after a sufficiently long time (i.e., as $t\rightarrow\infty$) the particle would be found in the first and second excited states of the infinite square well.

PROBLEM 63 – (Shankar 18.2.2)

A hydrogen atom is in its ground state at $t\rightarrow-\infty$. A time-dependent electric field of the form

$$\mathbf{E}(t) = \hat{\mathbf{z}}\mathcal{E}\exp(-t^2/\tau^2)$$

is applied till $t\rightarrow\infty$. Show that the probability of that the atom ends up in any of the $n=2$ states is, to first order, equal to:

$$P(n=2) = \left(\frac{2^{15}}{3^{10}}\right)\left(\frac{e\mathcal{E}a_0}{\hbar}\right)^2 \pi\tau^2\exp(-\omega^2\tau^2/2); \quad \Big(\omega\equiv(E_{21m}-E_{100})/\hbar\Big).$$