# QUANTUM MECHANICS B (PHY–5646)

## HOMEWORK 20

(March 21, 2017)
Due on Tuesday, March 28, 2017

### PROBLEM 58

We have calculated in class the second-order "Stark-Lo Surdo" (*i.e.*, dipole) shift in the ground-state energy of the hydrogen atom by including mixing with all states up to $n=3$, where $n$ denotes the principal quantum number. In this problem you will carry a "simpler" *first-order* calculation for the shift of all the $n=3$ states.

### PROBLEM 59 – (Shankar 17.3.2)

Consider a spin-1 particle with no orbital degrees of freedom. The Hamiltonian for such a particle in some arbitrary units of energy is given by:

$$\hat{H} = \frac{1}{\hbar^2}\left[\hat{S}_z^2 + \lambda\left(\hat{S}_x^2 - \hat{S}_y^2\right)\right] \equiv \hat{H}_0 + \lambda\hat{H}_1$$

where $\hat{S}_i$ are the three $3\times 3$ spin matrices and $\lambda \ll 1$. Treating the $\lambda$ term as a perturbation:

(a) Find the eigenstates of the unperturbed Hamiltonian $\hat{H}_0$ that are stable under the perturbation.

(b) Calculate the energy shifts to first order in $\lambda$.

(c) Obtain the exact eigenvalues of the full Hamiltonian $\hat{H}$ and show that they agree with the first-order estimate obtained in part (b) in the correct limit.

### PROBLEM 60

A system with an unperturbed Hamiltonian $\hat{H}_0$ is subject to a perturbation $\hat{H}_1$ where

$$\hat{H}_0 = E_0 \begin{pmatrix} 15 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \hat{H}_1 = E_0 \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

(a) Find the exact energies of the total Hamiltonian $\hat{H} = \hat{H}_0 + \lambda\hat{H}_1$.

(b) Find the eigenstates of the unperturbed Hamiltonian $\hat{H}_0$ that are stable under the perturbation.

(c) Calculate the eigenvalues of $\hat{H} = \hat{H}_0 + \lambda\hat{H}_1$ up to second order in $\lambda$.

(d) Show that to second order in $\lambda$ the results obtained in (a) and (c) are consistent with each other.