ELECTRICITY AND MAGNETISM II

Homework set #19: Potentials and Fields II and Radiation I

Problem # 19.1 :

A particle of charge q moves in a circle of radius a at constant angular velocity ω. Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at $(a, 0)$, on the positive x axis. Find the Liènard-Wiechert potentials for points on the z axis.

Problem # 19.2 :

Suppose a point charge q is constrained to move along the x axis. Show that the fields at points on the axis to the *right* of the charge are given by

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{1}{\mathcal{R}^2} \left(\frac{c+v}{c-v} \right) \hat{\mathbf{x}} \,, \quad \mathbf{B} = 0 \,.$$

Do not assume v is constant. What are the fields on the axis to the *left* of the charge?

Problem # 19.3 :

For a point charge moving at constant velocity, calculate the flux integral $\oint \mathbf{E} \cdot d\mathbf{a}$ over the surface of a sphere centered at the present location of the charge. Use that the electric field is given by

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1 - v^2/c^2}{[1 - v^2 \sin^2(\theta)/c^2]^{3/2}} \frac{\hat{\mathbf{R}}}{R^2} \,,$$

where $\mathbf{R} = \mathbf{r} - \mathbf{v}t$ is the vector from the present location to $\mathbf{r}$, and θ is the angle between $\mathbf{R}$ and $\mathbf{v}$.

Problem # 19.4 :

Find the **radiation resistance** of the wire joining the two ends of the electric dipole. This is the resistance that would give the same average power loss – to heat – as the oscillating dipole in fact puts out in the form of radiation.

(a) Show that $R = 790(d/\lambda)^2\Omega$, where λ is the wavelength of the radiation. (Ω = ohm).

(b) For the wires in an ordinary ratio (say, $d = 5$ cm) and $\lambda = 10^3$ m, should you worry about the radiative contribution to the total resistance?

Problem # 19.5 :

Check that the retarded potentials of an oscillating dipole (along the z-axis)

$$V(r,\theta,t) = \frac{p_0 \cos\theta}{4\pi\epsilon_0 r}\left\{-\frac{\omega}{c}\sin[\omega(t - r/c)] + \frac{1}{r}\cos[\omega(t - r/c)]\right\}$$

$$\mathbf{A}(r,\theta,t) = -\frac{\mu_0 p_0 \omega}{4\pi r}\sin[\omega(t - r/c)]\hat{\mathbf{z}}$$

satisfy the Lorentz gauge condition. Do not use approximation 3, $r \gg c/\omega$.