QUANTUM MECHANICS B (PHY–5646)

HOMEWORK 17

(February 13, 2017)

Due on Tuesday, February 21, 2017

PROBLEM 49

Consider three spin-1/2 particles (say an up, a down, and a strange quark).

(a) Using the law of addition of angular momentum, show that:

$$\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}=\frac{3}{2}\oplus\frac{1}{2}\oplus\frac{1}{2}.$$

(b) Construct using your method of choice, the eigenstates of good total angular momentum in terms of the direct product basis of the three particles.

(c) Identify the symmetry, if any, of the eigenstates of total angular momentum obtained in (b) under the exchange of any two particles.

PROBLEM 50

Consider two arbitrary vectors whose cartesian coordinates are given by:

$$\mathbf{A}=(A_x,A_y,A_z) \text{ and } \mathbf{B}=(B_x,B_y,B_z)\,.$$

(a) Obtain the spherical representation of the two vectors $\mathbf{A}=(A_+,A_0,A_-)$ and $\mathbf{B}=(B_+,B_0,B_-)$ by using the following unit vectors defined in class:

$$\hat{\mathbf{e}}_+\equiv-\frac{1}{\sqrt{2}}\Big(\hat{\mathbf{x}}+i\hat{\mathbf{y}}\Big),\ \hat{\mathbf{e}}_0\equiv\hat{\mathbf{z}},\ \text{and } \hat{\mathbf{e}}_-\equiv\frac{1}{\sqrt{2}}\Big(\hat{\mathbf{x}}-i\hat{\mathbf{y}}\Big).$$

(b) Build the rank-0 spherical tensor T_{00} from $\mathbf{A}$ and $\mathbf{B}$. Show that $T_{00}\propto\mathbf{A}\cdot\mathbf{B}$.

(c) Build the rank-1 spherical tensor T_{1q} from $\mathbf{A}$ and $\mathbf{B}$. Show that $T_{10}\propto(\mathbf{A}\times\mathbf{B})_z$.

(d) Build the rank-2 spherical tensor T_{2q} from $\mathbf{A}$ and $\mathbf{B}$.

PROBLEM 51

Consider the following symmetric, second rank tensor that one can build from the cartesian components of the spatial vector $\mathbf{r}$: $\hat{T}_{ij}=\hat{r}_i\hat{r}_j$.

(a) Using the results from Problem 50, construct all the component of the irreducible spherical tensor of rank 2.

(b) The *"quadrupole moment"* Q is defined as the following expectation value:

$$Q\equiv\langle\alpha,j,m=j\Big|(2\hat{z}^2-\hat{x}^2-\hat{y}^2)\Big|\alpha,j,m=j\rangle\,,$$

where α represents all quantum numbers needed to specify the state of the system other than those associated with the angular momentum. Using the Wigner-Eckart theorem, evaluate the following matrix element:

$$\langle\alpha,j,m'\Big|(\hat{x}^2-\hat{y}^2)\Big|\alpha,j,m\rangle\,,$$

in terms of Q and appropriate Clebsch-Gordan coefficients.