ELECTRICITY AND MAGNETISM II

Homework set #16: Electromagnetic Waves I

Problem # 16.1 :

Show that the **standing wave** $f(z,t) = A\sin(kz)\cos(kvt)$ satisfies the wave equation, and express it as the sum of a wave traveling to the left and a wave traveling to the right.

Problem # 16.2 :

Write down the real electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is

(a) traveling in the negative x direction and polarized in the z direction, and

(b) traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the xz plane. In each case, sketch the wave, and give the explicit Cartesian components of $\mathbf{k}$ and $\hat{\mathbf{n}}$.

Problem # 16.3 :

Find all the elements of the Maxwell stress tensor for a monochromatic plane wave traveling in the z direction. The plane wave is linearly polarized in the x direction. Does your answer make sense? Remember that $-\overleftrightarrow{\mathbf{T}}$ represents the momentum flux density. How is the momentum flux density related to the energy density, in this case?

Hint: The generic form for a plane wave is

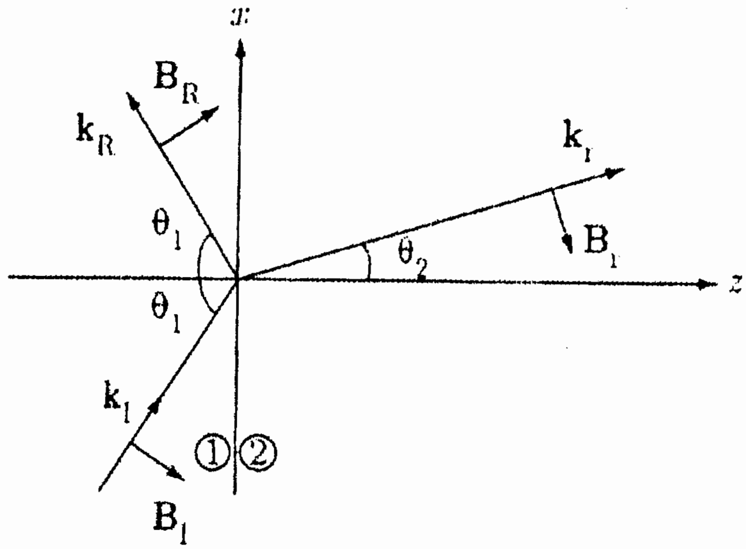
$$\tilde{\mathbf{E}}(\mathbf{r},t) = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\hat{\mathbf{n}} \ , \quad \tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}(\hat{\mathbf{k}}\times\hat{\mathbf{n}}) = \frac{1}{c}\hat{\mathbf{k}}\times\tilde{\mathbf{E}} \ ,$$

where $\hat{\mathbf{n}}$ is the polarization vector.

Problem # 16.4 :

In class we studied the case of polarization parallel to the plane of incidence, i.e. propagation and electric field in the xz plane. We now analyze the case of polarization *perpendicular* to the plane of incidence, i.e. the electric field is in the y direction, see figure.

(a) Impose the boundary conditions given below, and obtain the Fresnel equations for $\tilde{E}_{0R}$ and $\tilde{E}_{0T}$.



$$\text{(i)}\quad \epsilon_1\left(\tilde{\mathbf{E}}_{0_I}+\tilde{\mathbf{E}}_{0_R}\right)_z=\epsilon_2\left(\tilde{\mathbf{E}}_{0_T}\right)_z$$

$$\text{(ii)}\quad \left(\tilde{\mathbf{B}}_{0_I}+\tilde{\mathbf{B}}_{0_R}\right)_z=\left(\tilde{\mathbf{B}}_{0_T}\right)_z$$

$$\text{(iii)}\quad \left(\tilde{\mathbf{E}}_{0_I}+\tilde{\mathbf{E}}_{0_R}\right)_{x,y}=\left(\tilde{\mathbf{E}}_{0_T}\right)_{x,y}$$

$$\text{(iv)}\quad \frac{1}{\mu_1}\left(\tilde{\mathbf{B}}_{0_I}+\tilde{\mathbf{B}}_{0_R}\right)_{x,y}=\frac{1}{\mu_2}\left(\tilde{\mathbf{B}}_{0_T}\right)_{x,y}$$

(b) Sketch $\tilde{E}_{0R}/\tilde{E}_{0I}$ and $\tilde{E}_{0T}/\tilde{E}_{0I}$ as functions of θ_I, for the case $\beta = n_2/n_1 = 1.5$. Note that for this β the reflected wave is *always* 180^o out of phase.

(c) Show that there is no Brewster's angle for *any* n_1 and n_2: $\tilde{E}_{0R}$ is *never* zero (unless, of course, $n_1 = n_2$ and $\mu_1 = \mu_2$, in which case the two media are optically indistinguishable).

(d) Confirm that your Fresnel equations reduce to the proper forms at normal incidence.

(e) Compute the reflection and transmission coefficients, and check that they add up to 1.

Problem # 16.5 :

(a) Calculate the time-averaged energy density of an electromagnetic plane wave in a conducting medium. Use the *real* electric and magnetic fields. Show that the magnetic contribution always dominates. [*Answer:* $(k^2/2\mu\omega^2)E_0^2e^{-2\kappa z}$]

(b) Show that the intensity is $(k/2\mu\omega)E_0^2e^{-2\kappa\omega}$.