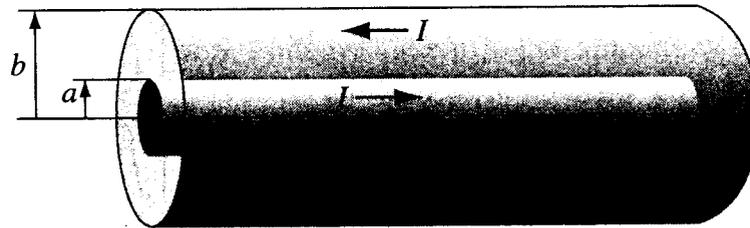


## ELECTRICITY AND MAGNETISM II

### Homework set #15: Conservation Laws

#### Problem # 15.1 :

- (a) Calculate the power (energy per unit time) transported down the coaxial cable, shown in the figure, consisting of an inner cylinder of radius  $a$  and an outer cylinder of radius  $b$ . Assume the two conductors are held at potential difference  $V$ , and carry current  $I$  (down one and back up the other).



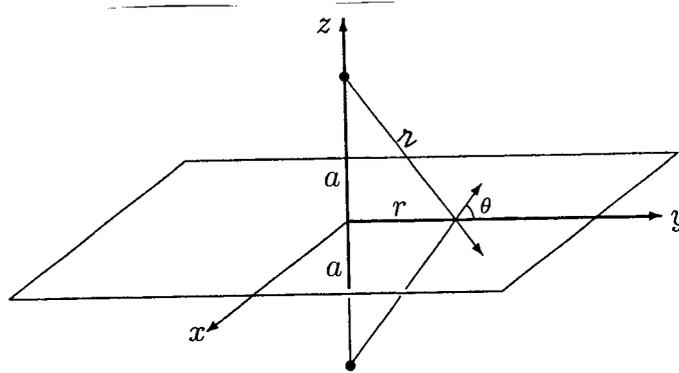
- (b) Consider now the charging capacitor of problem # 14.3.
- (i) Find the electric and magnetic fields in the gap, as functions of the distance  $s$  from the axis and the time  $t$ . (Assume the charge is zero at  $t = 0$ .)
- (ii) Find the energy density  $u_{em}$  and the Poynting vector  $\mathbf{S}$  in the gap. Note especially the *direction* of  $\mathbf{S}$ . Check that  $\partial u / \partial t = -\nabla \cdot \mathbf{S}$  is satisfied.
- (iii) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface. Check that the power input is equal to the rate of increase of energy in the gap,

$$\frac{d}{dt} \int_V u d\tau = - \oint_S \mathbf{S} \cdot d\mathbf{a} .$$

*Hint:* Ignore the fringing fields.

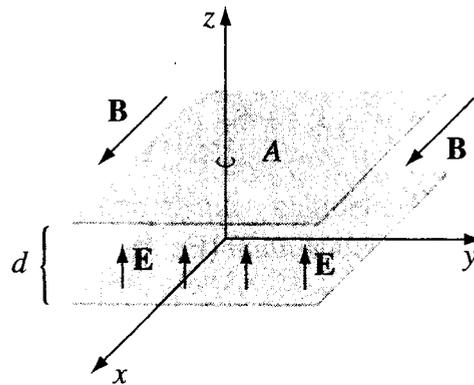
#### Problem # 15.2 :

- (a) Consider two equal point charges  $q$ , separated by a distance  $2a$ . Construct the plane equidistant from the two charges. By integrating Maxwell's stress tensor over this plane, determine the force of one charge on the other.
- (b) Do the same for charges that are opposite in sign.



**Problem # 15.3 :**

A charged parallel-plate capacitor (with uniform electric field  $\mathbf{E} = E\hat{z}$ ) is placed in a uniform magnetic field  $\mathbf{B} = B\hat{x}$ , as shown in the figure.



- (a) Find the electromagnetic momentum in the space between the plates.
- (b) Now a resistive wire is connected between the plates, along the  $z$  axis, so that the capacitor slowly discharges. The current through the wire will experience a magnetic force; what is the total impulse delivered to the system, during the discharge?

**Problem # 15.4 :**

Consider an infinite parallel-plate capacitor, with the lower plate (at  $z = -d/2$ ) carrying surface charge density  $-\sigma$ , and the upper plate (at  $z = +d/2$ ) carrying charge density  $+\sigma$ .

- (a) Determine all nine elements of the stress tensor, in the region between the plates.

Display your answer as a  $3 \times 3$  matrix:

$$\begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}.$$

- (b) Use  $\mathbf{F} = \oint_S \mathbf{T} \cdot d\mathbf{a}$  to determine the electromagnetic force per unit area on the top plate.
- (c) What is the electromagnetic momentum per unit area, per unit time, crossing the  $xy$  plane (or any other plane parallel to that one, between the plates)?
- (d) Of course, there must be *mechanical* forces holding the plates apart - perhaps the capacitor is filled with insulating material under pressure. Suppose we suddenly *remove* the insulator; the momentum flux (c) is now absorbed by the plates, and they begin to move. Find the momentum per unit time delivered to the top plate and compare your answer to (b).

*Note:* This is not an *additional* force, but rather an alternative way of calculating the same force - in (b) we got it from the force law, and in (d) we do it by conservation of momentum.

### Problem # 15.5 :

Two concentric spherical shells carry uniformly distributed charges  $+Q$  (at radius  $a$ ) and  $-Q$  (at radius  $b > a$ ). They are immersed in a uniform magnetic field  $\mathbf{B} = B_0 \hat{\mathbf{z}}$ .

- (a) Find the angular momentum of the fields (with respect to the center).
- (b) Now the magnetic field is gradually turned off. Find the torque on each sphere, and the resulting angular momentum of the system.