ELECTRICITY AND MAGNETISM II

Homework set #14: Electromagnetic Induction and Maxwell's Equations

Problem # 14.1 :

(a) Find the self-inductance per unit length of a long solenoid, of radius R, carrying n turns per unit length.

(b) Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length) using your result of part (a).

(c) Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length) using

$$W = \frac{1}{2}\oint (\mathbf{A}\cdot\mathbf{I})dl\ .$$

Note that the **A** has been worked out in an example in class.

(d) Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length) using

$$W = \frac{1}{2\mu_0}\int_{allspace} B^2 d\tau\ .$$

Problem # 14.2 :

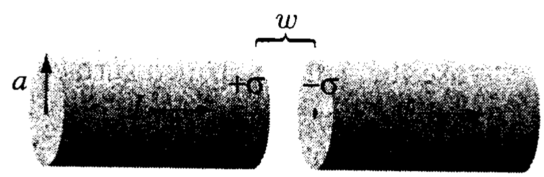
An infinite cylinder of radius R carries a uniform surface charge σ. We propose to set it spinning about its axis, at a final angular velocity ω_f. How much work will this take, per unit length? Do it two ways, and compare your answers:

(a) Find the magnetic field and the induced electric field (in the quasistatic approximation), inside and outside the cylinder, in terms of ω, $\dot{\omega}$, and s (the distance from the axis). Calculate the torque you must exert, and from that obtain the work done per unit length ($W = \int N d\phi$).

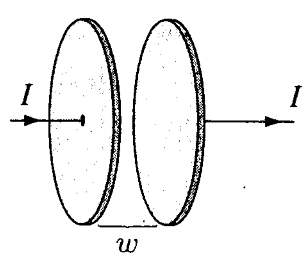
(b) Use $W = \frac{1}{2\mu_0} \int_{allspace} B^2 d\tau$ to determine the energy stored in the resulting magnetic field.

Problem # 14.3 :

(a) A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in the figure. Find the magnetic field in the gap, at a distance $s < a$ from the axis.



(b) For a more realistic model for a charging capacitor, imagine *thin* wires that connect to the centers of the plates (see figure). Again, the current I is constant, the radius of the capacitor is a, and the separation of the plates is $w \ll a$. Assume that the current flows out over the plates in such a way that the surface charge is uniform, at any time, and is zero at $t = 0$.



(i) Find the electric field between the plates, as a function of t.

(ii) Find the displacement current through a circle of radius s in the plane midway between the plates. Using this circle as your "Amperian loop", and the flat surface that spans it, find the magnetic field at a distance s from the axis.

Problem # 14.4 :

An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius a.

(a) In what *direction* does the induced electric field point (radial, circumferential or longitudinal)?

(b) Assuming that the field goes to zero as $s \to \infty$, find $\mathbf{E}(s,t)$.

Hint: The correct answer is

$$\mathbf{E}(s,t) = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \ln(a/s) \hat{\mathbf{z}} \ .$$

(c) Find the displacement current density $\mathbf{J}_d$.

(d) Integrate it to get the total displacement current,

$$I_d = \int \mathbf{J}_d \cdot d\mathbf{a} \ .$$

(e) Compare I_d and I. (What is their ratio?) If the outer cylinder is 2 mm in diameter, how high would the frequency have to be, for I_d to be 1% of I? [This problem is designed to indicate why Faraday never discovered displacement currents, and why it is ordinarily safe to ignore them unless the frequency is extremely high.]

Problem # 14.5 :

(a) Suppose there did exist magnetic monopoles. How would you modify Maxwell's equations and the force law to accommodate them? If you think there are several plausible options, list them, and suggest how you might decide experimentally which one is right.

(b) Assuming that "Coulomb's law" for magnetic charges (q_m) reads

$$\mathbf{F} = \frac{\mu_0}{4\pi} \frac{q_{m1} q_{m2}}{\mathcal{r}^2} \hat{\mathcal{r}},$$

work out the force law for a monopole q_m moving with velocity $\mathbf{v}$ through electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$.