

ELECTRICITY AND MAGNETISM II

Homework set #12: Electromotive Force

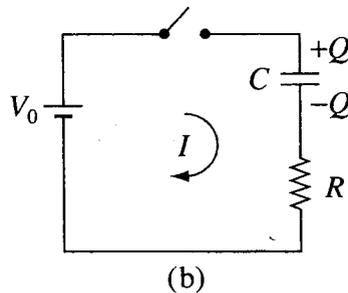
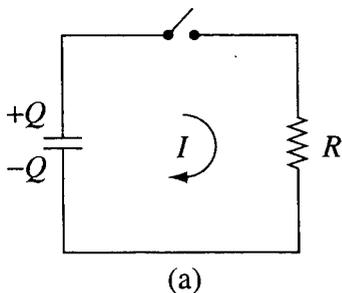
Problem # 12.1 :

A capacitor C has been charged up to potential V_0 ; at time $t = 0$, it is connected to a resistor R , and begins to discharge (see figure (a)).

- Determine the charge on the capacitor as a function of time, $Q(t)$. What is the current through the resistor, $I(t)$?
- What was the original energy stored in the capacitor, $W = \frac{1}{2}CV^2$? By integrating $P = I^2R$, confirm that the heat delivered to the resistor is equal to the energy lost by the capacitor.

Now imagine *charging up* the capacitor, by connecting it (and the resistor) to a battery of voltage V_0 , at time $t = 0$ (see figure (b)).

- Again determine $Q(t)$ and $I(t)$.
- Find the total energy output of the battery, i.e., $\int V_0 I dt$. Determine the heat delivered to the resistor. What is the final energy stored in the capacitor? What fraction of the work done by the battery shows up as energy in the capacitor? [Notice that the answer is independent of R !]

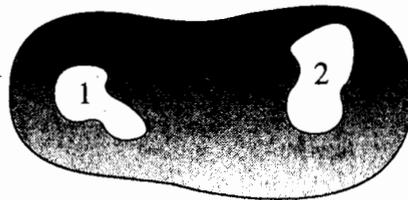


Problem # 12.2 :

- (a) Two metal objects are embedded in weakly conducting material of conductivity σ (see figure below). Show that the resistance between them is related to the capacitance of the arrangement by

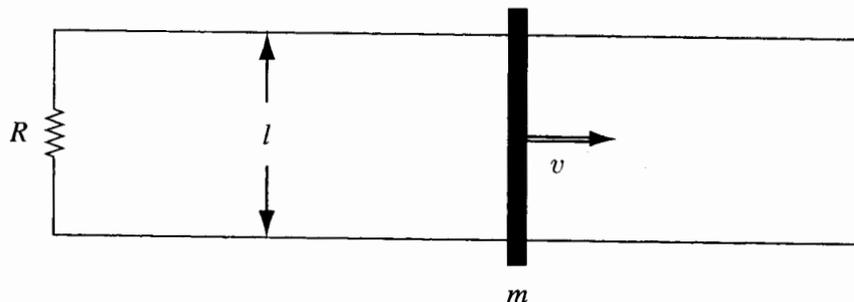
$$R = \frac{\epsilon_0}{\sigma C} .$$

- (b) Suppose you connected a battery between 1 and 2, and charged them up to a potential difference V_0 . If you then disconnect the battery, the charge will gradually leak off. Show that $V(t) = V_0 \exp(-t/\tau)$, and find the **time constant**, τ , in terms of ϵ_0 and σ .



Problem # 12.3 :

A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (see figure). A resistor R is connected across the rails, and a uniform magnetic field \mathbf{B} , pointing into the page, fills the entire region.

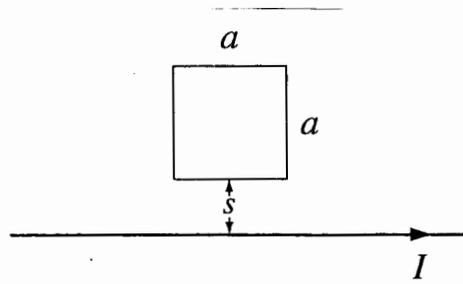


- (a) If the bar moves to the right at speed v , what is the current in the resistor? In what direction does it flow?

- (b) What is the magnetic force on the bar? In what direction?
- (c) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time?
- (d) The initial kinetic energy of the bar was, of course, $\frac{1}{2}mv_0^2$. Check that the energy delivered to the resistor is exactly $\frac{1}{2}mv_0^2$.

Problem # 12.4 :

A square loop of wire (side a) lies on a table, a distance s from a very long straight wire, which carries a current I , as shown in the figure below.



- (a) Find the flux of \mathbf{B} through the loop.
- (b) If someone now pulls the loop directly away from the wire, at speed v , what emf is generated? In what direction (clockwise or counterclockwise) does the current flow?
- (c) What if the loop is pulled to the *right* at speed v ?

Problem # 12.5 :

A long solenoid, of radius a , is driven by an alternating current, so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\hat{\mathbf{z}}$. A circular loop of wire, of radius $a/2$ and resistance R , is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.