QUANTUM MECHANICS A (PHY–5645)

HOMEWORK 1

(August 29, 2016)

Due on Tuesday, September 6, 2016

PROBLEM 1: Exercise 1.8.5 Shankar (almost!)

Consider the following matrix that represents a general rotation by an angle θ around the z-axis:

$$R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Show that the matrix $R_z(\theta)$ is unitary; note that in the case that all entries of the matrix are real the matrix is said to be *orthogonal*.

(b) Show that the eigenvalues of $R_z(\theta)$ are given by $e^{i\theta}$, $e^{-i\theta}$, and 1.

(c) Find the eigenvectors of $R_z(\theta)$ and show that they are orthogonal.

(d) Verify that the *transformed* matrix $U^\dagger R_z(\theta) U = \text{diag}(e^{i\theta}, e^{-i\theta}, 1)$ is the diagonal matrix of eigenvalues, where U is the matrix of eigenvectors, namely, the one that has the *normalized* eigenvectors of $R_z(\theta)$ as its columns.

PROBLEM 2

Euler's theorem for the rotation of a rigid body effectively says that any number of successive rotations are equivalent to a single rotation by a given angle along a given direction. In this problem you are going to verify Euler's theorem by considering the following two matrices that represent rotations by an angle $\pi/2$ along the x-axis and z-axis, respectively:

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad R_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Show that the matrix $R_{xz} = R_z \cdot R_x$ may be represented as a single rotation by explicitly finding the angle of rotation θ and the axis of rotation $\hat{\mathbf{n}}$.

(b) Do the same as in part (a), but now for the matrix $R_{zx} = R_x \cdot R_z$.

(c) Are the rotation angles found in parts (a) and (b) equal? Are the axes of rotation found in parts (a) and (b) equal? Explain your answer.

PROBLEM 3: Exercise 1.8.10 Shankar (almost!)

Consider the following two 3×3 symmetric matrices:

$$\Omega = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \quad \text{and} \quad \Lambda = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 2 \end{pmatrix} .$$

(a) Find the eigenvalues of both Ω and Λ.

(b) Show that the commutator of Ω and Λ vanishes. This implies that they share a common set of eigenvectors.

(c) Find the set of normalized eigenvectors common to both Ω and Λ.

(d) Find the unitary transformation U that brings Ω and Λ into diagonal form and verify that $U^\dagger \Omega U$ and $U^\dagger \Lambda U$ are the respective diagonal matrices of eigenvalues.

For this last part you should feel free to use Maple, Mathematica, MathLab, or any other symbolic program of your choice.