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Due 09/14/2016

Homework #2

1. Spaceship 1 passes spaceship 2 with a relative speed v. An observer in spaceship 1 measures a time interval Δt for spaceship 2 to pass by. Find the length of spaceship 2 as measured in its own rest frame, *i.e.*, find the proper length of spaceship 2 in terms of Δt.

$$L_1 = \frac{L_2}{\gamma} = v\Delta t \tag{1}$$

$$L_2 = \gamma L_1 = \gamma v \Delta t \tag{2}$$

2. A spaceship leaves earth destined for a neighboring star that is 4 light years away. The ship rapidly accelerates to a speed $v = 0.8c$ and then coasts.

a) How long will the trip take for an observer in the ship?

$$1 \text{ ly} = \frac{3\text{E+8 m}}{1 \text{ s}} \Bigg| \frac{60 \text{ s}}{1 \text{ m}} \Bigg| \frac{60 \text{ m}}{1 \text{ hr}} \Bigg| \frac{24 \text{ hr}}{1 \text{ d}} \Bigg| \frac{7 \text{ d}}{1 \text{ wk}} \Bigg| \frac{4 \text{ wk}}{1 \text{ mo}} \Bigg| \frac{12\text{mo}}{1 \text{ yr}} \tag{3}$$

$$\Delta x = \gamma \Delta x' \tag{4}$$

$$\frac{4}{\gamma} = \Delta x' \tag{5}$$

$$4\sqrt{1 - \left(\frac{0.8c}{c}\right)^2} = 2.4\text{ly} \tag{6}$$

$$\Delta t' = \frac{\Delta x}{v} = \frac{2.4}{0.8} = 3\text{yr} \tag{7}$$

b) What will the shipboard observer measure for the distance covered during the trip?

$$\Delta x' = v\Delta t' = 0.8(3) = 2.4\text{ly} \tag{8}$$

3. Consider the addition of velocities as shown in Eqs.(2.19)–(2.21). Find the x and y components of the velocity in the frame S of a light ray that has speed c along the y' axis of frame S' where S and S' are the usual frames we have been considering with S' moving with speed v with respect to S along the common $x - x'$ axes. Show that the light ray has speed c in the frame S.

$$x' = \gamma(x - vt), \qquad y' = y, \qquad t' = \gamma(t - \frac{v}{c^2}x)$$

$$u_x = \frac{\mathrm{d}x}{\mathrm{d}t}, \qquad u'_x = \frac{\mathrm{d}x'}{\mathrm{d}t'} = 0 \tag{9}$$

$$u'_x = \frac{\mathrm{d}x - v\mathrm{d}t}{\sqrt{1-\beta^2}} \frac{\sqrt{1-\beta^2}}{\mathrm{d}t - \frac{v}{c^2}\mathrm{d}x} = \frac{\mathrm{d}x - v\mathrm{d}t}{\mathrm{d}t - \frac{v}{c^2}\mathrm{d}x} \tag{10}$$

$$u'_x = \frac{\mathrm{d}t}{\mathrm{d}t}\left(\frac{\frac{\mathrm{d}x}{\mathrm{d}t} - v}{1 - \frac{v}{c^2}\frac{\mathrm{d}x}{\mathrm{d}t}}\right) = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} \tag{11}$$

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} = v \tag{12}$$

$$u'_y = \frac{\mathrm{d}y}{\gamma\left(\mathrm{d}t - \frac{v}{c^2}\mathrm{d}x\right)} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\gamma\left(1 - \frac{v}{c^2}\frac{\mathrm{d}x}{\mathrm{d}t}\right)} \tag{13}$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} = c\sqrt{1-\beta^2} \tag{14}$$

$$u = \sqrt{u_x^2 + u_y^2} = \sqrt{v^2 + c^2(1-\beta^2)} = \sqrt{v^2 + c^2 - v^2} = c \tag{15}$$

4. Consider a particle which is produced at $x = 0$ with a speed $v = 0.95c$ in the lab frame. The particle lives for a time $t = 2.2 \times 10^{-6}$ sec as measured in its rest frame before decaying.

a) How long does the particle live in the lab frame?

$$\gamma \Delta t = \Delta t'$$

$$\left[\sqrt{[1 - \left(\frac{0.95c}{c}\right)^2}\right]^{-1} (2.2\text{E-}6) = \Delta t' \tag{16}$$

$$\Delta t' = 7.05\mu s \tag{17}$$

b) How far does the particle travel in the lab frame?

$$d' = vt' = (7.05\text{E-}6)(0.95c) = 2009\text{m} \tag{18}$$

c) What is the distance between the production and decay locations in the lab as measured by someone in the particle rest frame?

$$\frac{1}{\gamma}\Delta x = \Delta x' = \sqrt{[1 - \left(\frac{0.95c}{c}\right)^2}(2009) = 627\text{m} \tag{19}$$

d) Show that observers in either frame will determine the relative speed of the two frames to be 0.95c.

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} = v = 0.95c, \qquad u'_x = 0 \tag{20}$$

$$u'_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} = \frac{0.95c + v}{1 + \frac{v}{c^2}0.95c} = 0 \tag{21}$$

$$v = -0.9c \qquad \therefore |v| = 0.95c \tag{22}$$

(5.) An observer measures the velocity of two electrons and finds that one has a speed $c/4$ along the x direction and the other has a speed $c/4$ in the y direction. Find the speed of the second electron as measured in the rest frame of the first electron.

$$u''_x = \frac{u_x - v}{1 - \frac{v}{c^2}u_x} = -v = -c/4 \qquad v = c/4, u_x = 0, u_y = c/4 \tag{23}$$

$$u''_y = \frac{u_y}{\gamma\left(1 - \frac{v}{c^2}u_x\right)} = \frac{c}{4\gamma} = \frac{c}{4}\sqrt{1 - (0.25)^2} \tag{24}$$

$$v'' = \left[\left(\frac{-c}{4}\right)^2 + \left(\frac{c}{4}\right)^2 \frac{3}{4}\right]^{1/2} = \frac{\sqrt{7}}{8}c \tag{25}$$